Math 105 TOPICS IN MATHEMATICS

SOLUTION FOR REGULAR HOMEWORK – XI (04/29)

May 4 (Mon), 2015

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[I] (16pts)

(1) $\sin 0 = 0.$ (2) $\sin \frac{\pi}{6} = \frac{1}{2}.$

(3) $\cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}.$ (4) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}.$

(5) $\cos \frac{\pi}{2} = 0.$ (6) $\cos \frac{5\pi}{6} = -\frac{\sqrt{3}}{2}.$

(7) $\sin\left(-\frac{\pi}{2}\right) = -1.$ (8) $\cos \pi = -1.$

[II] (4pts) (1) The distance $|PQ|$ between $P = (0,\ 0)$ and $Q = (5,\ 6)$.

[**Solution**]:

$$|PQ| = \sqrt{(5-0)^2 + (6-0)^2} = \sqrt{5^2 + 6^2} = \sqrt{61}\ .$$

(2) The distance $|PQ|$ between $P = (-4,\ 0)$, and $Q = (3,\ 1)$.

[**Solution**]:

$$|PQ| = \sqrt{\left(3 - (-4)\right)^2 + (1-0)^2} = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2}\ .$$

[III] (4pts) (1) What number does $\Big(\cos\theta\Big)^2 + \Big(\sin\theta\Big)^2$ equal?

[**Answer**]: 1.

(2) Paraphrase (1):

" The distance between

$$P = \Big(\boxed{\cos\theta}, \quad \boxed{\sin\theta} \Big)$$

and the coordinate origin $O = \big(0,\ 0\big)$ is always 1. "

[IV] (3pts) Let

$$P = \left(\cos\frac{2\pi}{5},\ \sin\frac{2\pi}{5}\right), \qquad Q = \left(\cos\frac{3\pi}{5},\ \sin\frac{3\pi}{5}\right),$$

$$R = \left(\cos\frac{\pi}{5},\ \sin\frac{\pi}{5}\right), \qquad S = \big(1,\ 0\big).$$

True or false : "$\big|PQ\big|$ and $\big|RS\big|$ are equal."

Explain.

[**Answer**]: True.

[**Explanation**]: P and Q are both lying in the unit circle such that the angle $\angle POQ$ equals $\frac{3\pi}{5} - \frac{2\pi}{5} = \frac{\pi}{5}$. R and S are also both lying in the unit circle such that the angle $\angle ROS$ equals $\frac{\pi}{5} - 0 = \frac{\pi}{5}$. Hence $\big|PQ\big|$ and $\big|RS\big|$ are naturally equal.

[V] (5pts) Assume

$$B_8{}^\circ(x) = x^8 - 4x^7 + \frac{14}{3}x^6 - \frac{7}{3}x^4 + \frac{2}{3}x^2,$$

and recover B_8 and $B_9{}^\circ(x)$. Follow the steps below:

(1) Add B_8 to $B_8{}^\circ(x)$. So,

$$B_8(x) = x^8 - 4x^7 + \frac{14}{3}x^6 - \frac{7}{3}x^4 + \frac{2}{3}x^2 + B_8.$$

(2) Take its antiderivative:

$$\int B_8(x)\,dx = \frac{1}{9}x^9 - \frac{1}{2}x^8 + \underline{\frac{2}{3}x^7 - \frac{7}{15}x^5 + \frac{2}{9}x^3} + B_8 \cdot x + C.$$

This is $\frac{1}{9}B_9{}^\circ(x)$. So

$$\frac{1}{9}B_9{}^\circ(x) = \frac{1}{9}x^9 - \frac{1}{2}x^8 + \underline{\frac{2}{3}x^7 - \frac{7}{15}x^5 + \frac{2}{9}x^3} + B_8 \cdot x + C.$$

(3) Substitute $x = 0$ and $x = 1$ into (2) independently. The outcomes are both 0. Thus

$$\begin{cases} 0 = \frac{1}{9}0^9 - \frac{1}{2}0^8 + \underline{\frac{2}{3}0^7 - \frac{7}{15}0^5 + \frac{2}{9}0^3} + B_8 \cdot 0 + C. \\ 0 = \frac{1}{9}1^9 - \frac{1}{2}1^8 + \underline{\frac{2}{3}1^7 - \frac{7}{15}1^5 + \frac{2}{9}1^3} + B_8 \cdot 1 + C. \end{cases}$$

The first of the two equations reads $0 = C$. So $C = 0$. Taking this into account, the second of the two equations becomes

$$0 = \frac{1}{9} - \frac{1}{2} \underline{\quad + \frac{2}{3} - \frac{7}{15} + \frac{2}{9} \quad} + B_8.$$

Solve it for B_8.

$$B_8 = -\frac{1}{30}.$$

(4) Substitute the value for B_8. Multiply 9 to the both sides. So

$$B_9{}^{\circ}(x) = x^9 - \frac{9}{2}x^8 \underline{\ + 6x^7 - \frac{21}{5}x^5 - 2x^3 - \frac{3}{10}x \ }.$$