## 

February 11 (Wed), 2015

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Line #: 52920.

[I] (6pts) (1) 
$$0^{12} = 0$$
. (2)  $1^{24} = 1$ . (3)  $(-1)^{99} = -1$ .

[II] (6pts) (1) 
$$\binom{11}{4} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{1 \cdot 2 \cdot 3 \cdot 4}$$
.

[III] (8pts) (a) 
$$(x + y)^5$$
  

$$= {5 \choose 0} x^5 + {5 \choose 1} x^4 y + {5 \choose 2} x^3 y^2 + {5 \choose 3} x^2 y^3 + {5 \choose 4} x y^4 + {5 \choose 5} y^5$$

$$= x^5 + 5 x^4 y + 10 x^3 y^2 + 10 x^2 y^3 + 5 x y^4 + y^5,$$

(b) 
$$(x+y)^6$$
  

$$= \begin{pmatrix} 6 \\ 0 \end{pmatrix} x^6 + \begin{pmatrix} 6 \\ 1 \end{pmatrix} x^5 y + \begin{pmatrix} 6 \\ 2 \end{pmatrix} x^4 y^2 + \begin{pmatrix} 6 \\ 3 \end{pmatrix} x^3 y^3 + \begin{pmatrix} 6 \\ 4 \end{pmatrix} x^2 y^4$$

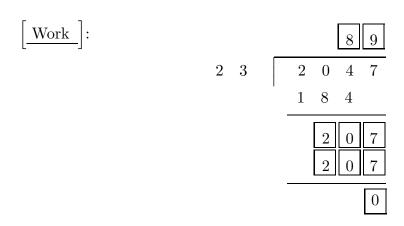
$$+ \begin{pmatrix} 6 \\ 5 \end{pmatrix} x y^5 + \begin{pmatrix} 6 \\ 6 \end{pmatrix} y^6$$

$$= x^6 + 6 x^5 y + 15 x^4 y^2 + 20 x^3 y^3 + 15 x^2 y^4 + 6 x y^5 + y^6$$

[IV] (10pts)

(1)  $2^{11} - 1$  is written as the product of two primes. One of the two primes is 23. What is the other prime?

Answer : 89.



(2) True or false: "If n is a prime, then  $2^n - 1$  is a prime."

 $[ \underline{ \text{Answer}} ]$ : False. (Part (1)  $(2^{11} - 1 = 23 \cdot 89)$  serves as a counterexample.)

(3) True or false: "If  $2^n - 1$  is a prime, then n is a prime."

Answer : True.

(4) <u>True or false</u>: "If  $2^n + 1$  is a prime, then n is a 2-to-the-power."

Answer : True.

(5) Is  $2^{32} + 1$  a prime? If not, what is the smallest prime that divides  $2^{32} + 1$ ?

<u>Answer</u>: 641.