Math 105 TOPICS IN MATHEMATICS

REVIEW OF LECTURES – XXXIII (SUPPLEMENT)

April 20 (Mon), 2015

Instructor: Yasuyuki Kachi

Line #: 52920.

Appendix to §33. System of linear Equations.

Can you solve the following for x and y?

$$\begin{cases} 3x + 2y = 4, & \text{— ①} \\ 7x + 5y = 1. & \text{— ②} \end{cases}$$

[**Solution**]: Do $-7 \cdot$ ①, and $3 \cdot$ ② each:

$$\begin{cases} -21x - 14y = -28, & \text{— ①}' \\ 21x + 15y = 3. & \text{— ②}' \end{cases}$$

Add ①′ and ②′ side by side:

$$y = -25.$$

Substitute this outcome into ①:

$$3x + 2 \cdot \big(-25\big) = 4.$$

Solve it for x:

$$3x = 4 - 2 \cdot \big(-25\big)$$

$$= 4 + 2 \cdot 25 = 54.$$

In short,

$$3x = 54.$$

So

$$x = 18.$$

So the answer is

$$x = 18, \qquad y = -25.$$

- The above example clearly falls into the pattern

$$\begin{cases} ax + by = p, & \text{— ①} \\ cx + dy = q, & \text{— ②} \end{cases}$$

with

$$a = 3, \qquad b = 2, \qquad p = 4,$$
$$c = 7, \qquad d = 5, \quad \text{and} \quad q = 1.$$

For these a, b, c and d we have

$$ad - bc = 1.$$

So, next let's solve

$$\begin{cases} ax + by = p, & \text{— ①} \\ cx + dy = q, & \text{— ②} \end{cases}$$

under the assumption $ad - bc = 1$.

Solution: Do $-c \cdot ①$, and $a \cdot ②$ each:

$$\begin{cases} -acx - bcy = -cp, & — ①' \\ acx + ady = aq. & — ②' \end{cases}$$

Add $①'$ and $②'$ side by side:

$$\Big(ad - bc\Big)y = -cp + aq.$$

Since by assumption $ad - bc = 1$, we obtain

$$y = -cp + aq.$$

Substitute this outcome into ①:

$$ax + b\Big(-cp + aq\Big) = p.$$

Solve it for x:

$$\begin{aligned} ax &= p - b\Big(-cp + aq\Big) \\ &= p + b \cdot cp - b \cdot aq \\ &= \Big(1 + b \cdot c\Big)p - a \cdot bq. \end{aligned}$$

Here, the assumption $ad - bc = 1$, can be paraphrased as $1 + bc = ad$, so the last quantity above further equals

$$adp - abq.$$

In short,

$$ax = adp - abq.$$

So

$$x = dp - bq.$$

So the answer is

$$x = dp - bq, \qquad y = -cp + aq.$$

Summary. The system of equations

$$\begin{cases} ax + by = p, & \text{— ①} \\ cx + dy = q, & \text{— ②} \end{cases}$$

under the assumption $\boxed{ad - bc = 1}$, is solved as

$$\boxed{x = dp - bq, \qquad y = -cp + aq}\,.$$

Exercise 1. Solve

(1) $\begin{cases} 2x + 3y = -1, \\ 3x + 5y = 3. \end{cases}$ (2) $\begin{cases} 11x - 14y = 6, \\ 4x - 5y = 2. \end{cases}$

[**Answers**]:

(1) $x = -14, \qquad y = 9.$

(2) $x = -2, \qquad y = -2.$