Math 290 ELEMENTARY LINEAR ALGEBRA

EXTRA CREDIT HOMEWORK – I

September 13 (Wed), 2017

Due Date: September 25 (Mon), 2017

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Line #: 25751.

ID # : ____________________ Name : ____________________________

[I] (20pts) Compare the following two systems of equations:

$$
(*) \quad \begin{aligned}
x_1x_6 - x_2x_5 + x_3x_8 - x_4x_7 &= 1, & x_9x_{14} - x_{10}x_{13} + x_{11}x_{16} - x_{12}x_{15} &= 1,\\
x_1x_{10} - x_2x_9 + x_3x_{12} - x_4x_{11} &= 0, & x_5x_{14} - x_6x_{13} + x_7x_{16} - x_8x_{15} &= 0,\\
x_1x_{14} - x_2x_{13} + x_3x_{16} - x_4x_{15} &= 0, & x_5x_{10} - x_6x_9 + x_7x_{12} - x_8x_{11} &= 0,\\
x_1x_8 - x_4x_5 + x_2x_7 - x_3x_6 &= 0, & x_9x_{16} - x_{12}x_{13} + x_{10}x_{15} - x_{11}x_{14} &= 0,\\
x_1x_{12} - x_4x_9 + x_2x_{11} - x_3x_{10} &= 0, & x_5x_{16} - x_8x_{13} + x_6x_{15} - x_7x_{14} &= 0,\\
x_1x_{16} - x_4x_{13} + x_2x_{15} - x_3x_{14} &= 1, & x_5x_{12} - x_8x_9 + x_6x_{11} - x_7x_{10} &= 1,\\
x_1x_7 - x_3x_5 - x_2x_8 + x_4x_6 &= 0, & x_9x_{15} - x_{11}x_{13} - x_{10}x_{16} + x_{12}x_{14} &= 0,\\
x_1x_{11} - x_3x_9 - x_2x_{12} + x_4x_{10} &= 1, & x_5x_{15} - x_7x_{13} - x_6x_{16} + x_8x_{14} &= -1,\\
x_1x_{15} - x_3x_{13} - x_2x_{16} + x_4x_{14} &= 0, & x_5x_{11} - x_7x_9 - x_6x_{12} + x_8x_{10} &= 0,
\end{aligned}
$$

and

$$
(**) \quad \begin{aligned}
-x_1 - x_6 &= 0, & x_4 - x_{10} &= 0, & -x_3 - x_{14} &= 0,\\
x_8 + x_9 &= 0, & -x_7 + x_{13} &= 0, & -x_{11} - x_{16} &= 0,\\
x_3 - x_8 &= 0, & -x_2 - x_{12} &= 0, & -x_1 - x_{16} &= 0,\\
-x_6 - x_{11} &= 0, & -x_5 - x_{15} &= 0, & -x_9 + x_{14} &= 0,\\
-x_4 - x_7 &= 0, & -x_1 - x_{11} &= 0, & x_2 - x_{15} &= 0,\\
-x_5 + x_{12} &= 0, & x_6 + x_{16} &= 0, & x_{10} + x_{13} &= 0.
\end{aligned}
$$

(1) Find the complete solution set for the system $(**)$:

$$\begin{pmatrix} x_1, & x_2, & x_3, & x_4, \\ x_5, & x_6, & x_7, & x_8, \\ x_9, & x_{10}, & x_{11}, & x_{12}, \\ x_{13}, & x_{14}, & x_{15}, & x_{16} \end{pmatrix} = \begin{pmatrix} \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_, \\ \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_, \\ \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_, \\ \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_ \end{pmatrix}.$$

The right way to describe the solution set is to parametrize it. Thus, your answer should involve parameters (such as s, t, u, *etc.*).

(2) Find the exact condition (= necessary and sufficient condition) for the parametrized solution which you found in (1) to also become a solution for $(*)$. The answer is a single identity that involves the parameters which you have used in your answer for for (1) (s, t, u, *etc.*).

______________________________________ .

(3) Give your <u>wild guess</u> of the complete solution set <u>for $(*)$</u>.

$$\begin{pmatrix} x_1, & x_2, & x_3, & x_4, \\ x_5, & x_6, & x_7, & x_8, \\ x_9, & x_{10}, & x_{11}, & x_{12}, \\ x_{13}, & x_{14}, & x_{15}, & x_{16} \end{pmatrix} = \begin{pmatrix} \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_, \\ \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_, \\ \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_, \\ \_\_\_, & \_\_\_, & \_\_\_, & \_\_\_ \end{pmatrix},$$

such that ______________________________________ (<u>constraint</u>).

Again, the right way to describe the solution set is to parametrize it. Thus, your answer should involve parameters (such as s, t, u, v *etc.*). Since this is a "guess", no justification is necessary. Note that, the answer for (3) and the answer for (1) are <u>not identical</u>.