Math 290 ELEMENTARY LINEAR ALGEBRA

MIDTERM EXAM (In-Class)

October 9 (Mon), 2017

Instructor: Yasuyuki Kachi

Line #: 25751.

ID # : ______________ Name : ______________________

This in-class exam is worth 120 points. The duration of this exam is 50 minutes (start at 11:00am, finish at 11:50am).

[I] (20pts) (1) What is the 2×2 identity matrix I? Write it out as in

$$I = \begin{bmatrix} \Box & \Box \\ \Box & \Box \end{bmatrix}.$$

(2) The system

$$\begin{cases} 2x - 7y = 5, \\ x - 6y = 1 \end{cases}$$

is rewritten as

$$\underbrace{\begin{bmatrix} \Box & \Box \\ \Box & \Box \end{bmatrix}}_{\substack{\| \\ A}} \underbrace{\begin{bmatrix} \Box \\ \Box \end{bmatrix}}_{\substack{\| \\ \boldsymbol{x}}} = \underbrace{\begin{bmatrix} \Box \\ \Box \end{bmatrix}}_{\substack{\| \\ \boldsymbol{b}}}.$$

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[II] (20pts) Let

$$A = \begin{bmatrix} 5 & 2\sqrt{6} \\ 2\sqrt{6} & 5 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 9 & 4\sqrt{5} \\ 4\sqrt{5} & 9 \end{bmatrix}.$$

(1a) $\det A =$ __________ .

(1b) $\det B =$ __________ .

(2) $\det\Big(AB\Big) =$ __________ .

(3) True or false : $\det\Big(AB\Big) = \Big(\det A\Big)\Big(\det B\Big)$.

☐ True. ☐ False. (Check one.)

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[III] (40pts) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$. Suppose $AB = I$.

(1) True or false : $BA = I$.

☐ True. ☐ False. (Check one.)

(2) True or false : $B = A^{-1}$.

☐ True. ☐ False. (Check one.)

(3) True or false : $A = B^{-1}$.

☐ True. ☐ False. (Check one.)

(4) Explain why you chose the answers you chose for (1), (2) and (3) above.

Reason: If $AB = I$, then by ______________ rule, $\det A$ ____.

So ☐ exists. Multiply ☐ from the ______ to the two sides

of ☐ and get $B = A^{-1}$, thanks to ______________ law.

Then $BA = I$. Hence also $A = B^{-1}$.

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[IV] (10pts) True or False :

For

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad \text{and} \qquad B = \begin{bmatrix} p & q \\ r & s \end{bmatrix},$$

(1) $$\Big(A + B\Big)\Big(A - B\Big) = A^2 - B^2.$$

☐ True. ☐ False. (Check one.)

(2) $$\Big(\det A\Big) + \Big(\det B\Big) = \det \Big(A + B\Big).$$

☐ True. ☐ False. (Check one.)

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[V] (10pts) Find the determinant of

$$A = \begin{bmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{bmatrix}.$$

$\det A =$ ____________ .

Work.

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[VI] (20pts) (1) Circle all matrices that are in reduced row echelon form:

(1a) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$. (1b) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

(1c) $\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. (1d) $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

(1e) $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$. (1f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(2) Make the matrix below a reduced row echelon form, by filling in either 0 or '$*$' (where $*$ denotes an arbitrary number) to the empty boxes, where all the leading 1s are already shown in the matrix.

$$\begin{bmatrix} \boxed{1} & \boxed{} & \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{1} & \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{1} & \boxed{} & \boxed{} & \boxed{} \end{bmatrix}.$$

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[VII] (Extra Credit; 10pts) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

Assume $A^T = -A$. Prove that there is a scalar s such that

$$A = s\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}.$$

Proof. $A^T = -A$ reads

$$\underbrace{\begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}}_{\substack{\| \\ A^T}} = \underbrace{\begin{bmatrix} \boxed{-a} & \boxed{-b} \\ \boxed{-c} & \boxed{-d} \end{bmatrix}}_{\substack{\| \\ -A}}.$$

Hence

$$\boxed{} = -a, \qquad \boxed{} = -b,$$
$$\boxed{} = -c, \qquad \boxed{} = -d.$$

From these, conclude

$a =$ ______ , $d =$ ______ , $b =$ __________ .

Hence

$$A = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix} = \boxed{} \begin{bmatrix} \boxed{0} & \boxed{} \\ \boxed{} & \boxed{0} \end{bmatrix}.$$