Math 290 ELEMENTARY LINEAR ALGEBRA

FINAL EXAM (Take-home)

December 6 (Wed), 2017

Due date: December 15 (Fri), 2017

Instructor: Yasuyuki Kachi

Line #: 25751.

ID # : ____________________ Name : ____________________________

This take-home part of Midterm Exam is worth 140 points and is due in class Friday, December 15th, 2017. Submission after 1 : 30 pm, December 15th, will not be accepted.

[I] (Take-home; 20pts) Let A be an $n \times n$ matrix, with entires in $\mathbb{C}$. Let $\lambda = \lambda_0 \in \mathbb{C}$ be one of the eigenvalues of A. Recall that the eigenspace of A with respect to its eigenvalue $\lambda = \lambda_0$ is defined as

$$V_{\lambda_0} = \left\{ \boldsymbol{x} \in \mathbb{C}^n \;\middle|\; A\boldsymbol{x} = \lambda_0 \boldsymbol{x} \right\}.$$

(1) True or false : "$\boldsymbol{x} \in V_{\lambda_0}, \ \boldsymbol{y} \in V_{\lambda_0} \implies \boldsymbol{x} + \boldsymbol{y} \in V_{\lambda_0}$."

☐ True. ☐ False. (Check one.)

(2) True or false : "$\boldsymbol{x} \in V_{\lambda_0}, \ \alpha \in \mathbb{C} \implies \alpha \boldsymbol{x} \in V_{\lambda_0}$."

☐ True. ☐ False. (Check one.)

Line #: 25751.

ID # : ____________________ Name : ____________________

[II] (Take-home; 30pts) Complete the field axioms.

- **Field axioms.** Each of $k = \mathbb{R}$ and $k = \mathbb{C}$ satisfies the following axioms:

(i) $\alpha + \beta \;=\; \underline{\quad \beta + \alpha \quad}$.

(ii) $\alpha + \Big(\beta + \gamma\Big) \;=\; \underline{\Big(\qquad\qquad \Big) + \qquad\qquad}$.

(iii) $\alpha + 0 \;=\; \underline{\qquad\quad}$.

(iv) $\alpha + \Big(-\alpha\Big) \;=\; \underline{\qquad\quad}$.

(v) $\alpha\beta \;=\; \underline{\quad \beta\alpha \quad}$.

(vi) $\alpha\Big(\beta\gamma\Big) \;=\; \underline{\Big(\qquad\quad \Big) \qquad\quad}$.

(vii) $\Big(\alpha + \beta\Big)\gamma \;=\; \underline{\qquad\qquad + \qquad\qquad}$.

(viii) $\alpha \cdot 1 \;=\; \underline{\qquad\quad}$.

(ix) For $\alpha \neq 0$, there is α^{-1} such that $\alpha\alpha^{-1} = 1$.

(x) $0 \neq 1$.

Line #: 25751.

ID # : ____________________ Name : ____________________

[III] (Take-home; 30pts) Let k be a field.

(1) True or false : "In a field k, $0 \cdot \alpha = 0$."

☐ True. ☐ False. (Check one.)

(2) Give the definition of $2 \in k$.

$$2 = \underline{\qquad + \qquad}.$$

(3) True or false : "In a field k, $1 + 1 \neq 0$."

☐ Always true, no matter what the field k is.

☐ Not always true. It depends on the field k.

(Check one.)

Line #: 25751.

ID # : ______________________   Name : ______________________________

[IV] (Take-home; 30pts) Complete the vector space field axioms.

- **Vector space axioms.**

Let k be a field. V is said to be a k-vector space, if $\boldsymbol{x}+\boldsymbol{y} \in V$ $\Big(\boldsymbol{x}, \boldsymbol{y} \in V\Big)$ and $\alpha\boldsymbol{x} \in V$ $\Big(\alpha \in k$ and $\boldsymbol{x} \in V\Big)$ are both defined, and furthermore, V has a distinguishable element $\mathbf{0}$, such that the following (i) through (viii) are satisfied:

(i) $\boldsymbol{x}+\boldsymbol{y} = \underline{\quad \boldsymbol{y}+\boldsymbol{x} \quad}$.

(ii) $\boldsymbol{x}+\Big(\boldsymbol{y}+\boldsymbol{z}\Big) = \underline{\Big(\qquad \Big) + \qquad}$.

(iii) $\boldsymbol{x}+\mathbf{0} = \underline{\qquad}$.

(iv) For each $\boldsymbol{x} \in V$, there exists $-\boldsymbol{x} \in V$ such that $\boldsymbol{x}+\Big(-\boldsymbol{x}\Big) = \underline{\qquad}$.

(v) $\alpha\Big(\beta\boldsymbol{x}\Big) = \underline{\Big(\qquad \Big)}$.

(vi) $\alpha\Big(\boldsymbol{x}+\boldsymbol{y}\Big) = \underline{\qquad + \qquad}$.

(vii) $\Big(\alpha+\beta\Big)\boldsymbol{x} = \underline{\qquad + \qquad}$.

(viii) $1 \cdot \boldsymbol{x} = \underline{\qquad}$.

Line #: 25751.

ID # : ____________________ Name : ____________________

[V] (Take-home; 30pts) Let k be a field.

(1) True or false : "k itself is a k-vector space."

☐ True. ☐ False. (Check one.)

(2) True or false : "$\{\mathbf{0}\}$ is a k-vector space."

☐ True. ☐ False. (Check one.)

(3)

$$k^n = \left\{ \boldsymbol{x} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \;\middle|\; \alpha_1,\ \alpha_2,\ \cdots,\ \alpha_n \in \boxed{} \right\}$$

is an example of a k-vector space, with respect to the usual vector addition and scalar multiplication. (Fill an appropriate symbol in the box.)