Math 290 ELEMENTARY LINEAR ALGEBRA

PRACTICE MIDTERM (for In-Class)

October 2 (Mon), 2017

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Line #: 25751.

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⋆ **The actual exam may not be very similar to this practice exam. The purpose of this practice exam is to give you an idea of how the actual exam will look like, in terms of the length and the format. This practice exam is for the "in-class" portion of the exam only.**

[I] (20pts) (1) What is the 2×2 identity matrix I? Write it out as in

$$I = \begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}.$$

(2) The system

$$\begin{cases} 3x - 2y = 6, \\ 5x + y = -4 \end{cases}$$

is rewritten as

$$\underbrace{\begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}}_{\substack{\| \\ A}} \underbrace{\begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}}_{\substack{\| \\ \boldsymbol{x}}} = \underbrace{\begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}}_{\substack{\| \\ \boldsymbol{b}}}.$$

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([I] continued)

(3) Solve the system in (2) in the previous page.

$$\boldsymbol{x} = A^{-1}\boldsymbol{b} = \overbrace{\frac{1}{\boxed{}}\begin{bmatrix} \boxed{} & \boxed{} \\ \boxed{} & \boxed{} \end{bmatrix}}^{\substack{A^{-1} \\ \|}} \overbrace{\begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}}^{\substack{\boldsymbol{b} \\ \|}}$$

$$= \frac{1}{\boxed{}} \begin{bmatrix} \boxed{} \\ \boxed{} \end{bmatrix}.$$

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[II] (20pts) Let

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 2 \end{bmatrix} \qquad \text{and} \qquad B = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}.$$

(1a) $\det A =$ __________ .

(1b) $\det B =$ __________ .

(2) $\det \Big(AB \Big) =$ __________ .

(3) <u>True or false</u> : $\det \Big(AB \Big) = \Big(\det A \Big) \Big(\det B \Big).$

☐ True. ☐ False. (<u>Check one.</u>)

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[III] (30pts) (1) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

(1a) True or false : $IA = A$.

☐ True. ☐ False. (Check one.)

(1b) True or false : $AI = A$.

☐ True. ☐ False. (Check one.)

(2) For A as in (1), suppose $ad - bc \neq 0$.

(2a) True or false : $AA^{-1} = I$.

☐ True. ☐ False. (Check one.)

(2b) True or false : $A^{-1}A = I$.

☐ True. ☐ False. (Check one.)

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([III] continued)

(3) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$. Suppose $AB = I$.

(3a) <u>True or false</u> : $BA = I$.

☐ True. ☐ False. (<u>Check one.</u>)

(3b) <u>True or false</u> : $B = A^{-1}$.

☐ True. ☐ False. (<u>Check one.</u>)

(3c) <u>True or false</u> : $A = B^{-1}$.

☐ True. ☐ False. (<u>Check one.</u>)

(3d) Explain why you chose the answers you chose for (3a), (3b) and (3c) above.

Reason: If $AB = I$, then by ________________ rule, $\det A$ ______.

So $\boxed{}$ exists. Multiply $\boxed{}$ from the ________ to the two sides

of $\boxed{}$ and get $B = A^{-1}$, thanks to ________________ law.

By (2b), $BA = I$. Hence also $A = B^{-1}$.

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[IV] (10pts) True or False :

For

$$A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}, \quad B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix}, \quad C = \begin{bmatrix} a_3 & b_3 \\ c_3 & d_3 \end{bmatrix}, \quad D = \begin{bmatrix} a_4 & b_4 \\ c_4 & d_4 \end{bmatrix},$$

(1a) $$A\Big(B+C\Big) = AB + AC.$$

☐ True. ☐ False. (Check one.)

(1b) $$\Big(B+C\Big)D = BD + CD.$$

☐ True. ☐ False. (Check one.)

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[V] (10pts) Find the determinant of

$$A = \begin{bmatrix} 0 & -c & b \\ c & 0 & -a \\ -b & a & 0 \end{bmatrix}.$$

$$\det A = \underline{\hspace{3cm}}.$$

Work.

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[VI] (20pts) (1) Circle all matrices that are in reduced row echelon form:

(1a) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$ (1b) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$

(1c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$ (1d) $\begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}.$

(1e) $\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$ (1f) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$

(2) Make the matrix below a reduced row echelon form, by filling in either 0 or '$*$' (where $*$ denotes an arbitrary number) to the empty boxes, where all the leading 1s are already shown in the matrix.

$$\begin{bmatrix} \boxed{1} & \boxed{} & \boxed{} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{1} & \boxed{} & \boxed{} & \boxed{} \\ \boxed{} & \boxed{} & \boxed{} & \boxed{} & \boxed{} & \boxed{1} \end{bmatrix}.$$

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[VII] (10pts) Let

$$A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \qquad B = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix},$$

$$C = \begin{bmatrix} \cos\Big(\theta+\phi\Big) & -\sin\Big(\theta+\phi\Big) \\ \sin\Big(\theta+\phi\Big) & \cos\Big(\theta+\phi\Big) \end{bmatrix}.$$

(1) Write out AB <u>in terms of $\sin\theta$, $\cos\theta$, $\sin\phi$ and $\cos\phi$</u> .

AB

$$= \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{}\cdot\boxed{} - \boxed{}\cdot\boxed{} & -\Big(\boxed{}\cdot\boxed{} + \boxed{}\cdot\boxed{}\Big) \\ \boxed{}\cdot\boxed{} + \boxed{}\cdot\boxed{} & \boxed{}\cdot\boxed{} - \boxed{}\cdot\boxed{} \end{bmatrix}.$$

(2) Know $C = AB$. Write out its consequence:

$$\cos\Big(\theta+\phi\Big) = \boxed{}\cdot\boxed{} - \boxed{}\cdot\boxed{},$$

$$\sin\Big(\theta+\phi\Big) = \boxed{}\cdot\boxed{} + \boxed{}\cdot\boxed{}.$$

$\star$ Acknowledge that the two equations in (2) are the addition formulas for trigs.

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[VIII] (10pts) Find the <u>characteristic polynomial</u>, and the <u>eigenvalues</u>, of the matrix A below:

$$A = \begin{bmatrix} 6 & 1 \\ 2 & 7 \end{bmatrix}.$$