Math 290 ELEMENTARY LINEAR ALGEBRA
PRACTICE FINAL (for In-Class)

December 9 (Wed), 2017

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⋆ **The actual exam may not be very similar to this practice exam. The purpose of this practice exam is to give you an idea of how the actual exam will look like, in terms of the length and the format. This practice exam is for the "in-class" portion of the exam only.**

[I] (20pts)

(1) $$\det \begin{bmatrix} 4 & 0 & 1 \\ 3 & 0 & 2 \\ 2 & 0 & 3 \end{bmatrix} = \underline{\qquad\qquad} .$$

(2) $$\det \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \underline{\qquad\qquad} .$$

(3) $$\det \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} = \underline{\qquad\qquad} .$$

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([I] continued)

(4)
$$\begin{vmatrix} 1\cdot 1 & 1\cdot 2 & 1\cdot 3 \\ 2\cdot 1 & 2\cdot 2 & 2\cdot 3 \\ 3\cdot 1 & 3\cdot 2 & 3\cdot 3 \end{vmatrix} = \underline{\qquad\qquad} .$$

(5)
$$\begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{vmatrix} = \underline{\qquad\qquad} .$$

(6)
$$\begin{vmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{vmatrix} = \underline{\qquad\qquad} .$$

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[II] (20pts)

(1) Observe

$$A = \begin{bmatrix} a_{11} & \boxed{a_{12}} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & \boxed{a_{24}} \\ \boxed{a_{31}} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & \boxed{a_{43}} & a_{44} \end{bmatrix}.$$

$a_{12}\ a_{24}\ a_{31}\ a_{43}$

☐ is an elementary product.

☐ is not an elementary product.

(Check one.)

(2) Observe

$$A = \begin{bmatrix} \boxed{a_{11}} & a_{12} & a_{13} & a_{14} \\ a_{21} & \boxed{a_{22}} & a_{23} & a_{24} \\ a_{31} & a_{32} & \boxed{a_{33}} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}.$$

$a_{11}\ a_{22}\ a_{33}$

☐ is an elementary product.

☐ is not an elementary product.

(Check one.)

(3) The number of elementary products of the general 4×4 matrix A

$$= \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{1} = \boxed{}.$$

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[III] (8pts) (1) The number of inversions for the permutation

$$\left(4,\ 2,\ 5,\ 3,\ 1 \right)$$

is calculated as

$$\boxed{} + \boxed{} + \boxed{} + \boxed{} = \boxed{}\ .$$

contribution from $\boxed{4}$	contribution from $\boxed{2}$	contribution from $\boxed{5}$	contribution from $\boxed{3}$	total

(2) Hence, the permutation in (1) is ______________ .

(Fill in either "even" or "odd" .)

(3) In the formula defining the determiant

$$\det \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix},$$

☐ $+\, a_{14}\, a_{22}\, a_{35}\, a_{43}\, a_{51}$

☐ $-\, a_{14}\, a_{22}\, a_{35}\, a_{43}\, a_{51}$

appears as a term. (Check one.)

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[IV] (12pts)

(1) $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

$$= 1 \cdot \begin{vmatrix} b & c \\ b^2 & c^2 \end{vmatrix} - a \cdot \begin{vmatrix} 1 & 1 \\ b^2 & c^2 \end{vmatrix} + a^2 \cdot \begin{vmatrix} 1 & 1 \\ b & c \end{vmatrix} \quad \Big(\text{co-factor expansion}\Big)$$

$$= 1 \cdot \left(\boxed{} - \boxed{} \right)$$

$$- a \cdot \left(\boxed{} - \boxed{} \right)$$

$$+ a^2 \cdot \left(\boxed{} - \boxed{} \right)$$

$$= bc \cdot \left(\boxed{} - \boxed{} \right) - a\Big(c + b\Big)\left(\boxed{} - \boxed{} \right)$$

$$+ a^2 \left(\boxed{} - \boxed{} \right)$$

$$= \left[bc - \Big(c + b\Big)a + a^2 \right] \left(\boxed{} - \boxed{} \right)$$

$$= \left(\boxed{b} - \boxed{a} \right) \left(\boxed{} - \boxed{} \right) \left(\boxed{} - \boxed{} \right).$$

(2) $\begin{vmatrix} 1 & 1 & 1 \\ 1 & 4 & 9 \\ 1 & 16 & 81 \end{vmatrix} =$ __________ .

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[V] (20pts) (1) Give the definition of an <u>orthogonal</u> matrix.

" A square matrix A is said to be <u>orthogonal</u>, when

________________________ holds. "

(2) For an arbitrary orthogonal matrix A (with real numbers as entries),

$$\det A = \underline{\qquad\qquad}, \quad \text{or} \quad \underline{\qquad\qquad}.$$

(3) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is <u>orthogonal</u>, and $ad - bc = -1$, then

$$A^2 = \underline{\qquad\quad}.$$

Proof for (3): A is written as $A = \begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, with $a^2 + b^2 = 1$.

Accordingly,

$$A^2 = \begin{bmatrix} a & b \\ b & -a \end{bmatrix} \begin{bmatrix} a & b \\ b & -a \end{bmatrix} = \begin{bmatrix} \boxed{\qquad\qquad} & \boxed{\qquad\qquad} \\ \boxed{\qquad\qquad} & \boxed{\qquad\qquad} \end{bmatrix}$$

$$= \begin{bmatrix} \boxed{\quad} & \boxed{\quad} \\ \boxed{\quad} & \boxed{\quad} \end{bmatrix} \qquad \left[\text{by } a^2 + b^2 = 1.\right]$$

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[VI] (20pts) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. (1) True or false :

(1a) $A + A^T$ is a symmetric matrix.

☐ True. ☐ False. (Check one.)

(1b) AA^T is a symmetric matrix.

☐ True. ☐ False. (Check one.)

(1c) $AA^T = A^T A$.

☐ True. ☐ False. (Check one.)

(2) Let

$$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}.$$

Agree that this is a symmetric matrix. Suppose a, b, c and d are real numbers.

True or false :

"*A is diagonalized by an orthogonal matrix. Namely, there exists an orthogonal matrix Q such that $Q^{-1}AQ$ is a diagonal matrix.*"

☐ True. ☐ False. (Check one.)

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[VII] (20pts) Diagonalize the given matrix A. If not feasible, then say 'not feasible'.

$$A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}.$$